343. (*Selected*) *by Prof. H. T. Eddy.*—If E^2 be the sum of the squares of the edges of a tetrahedron, F^2 the sum of the squares of the areas of the faces and V the volume, show that the principal semi axes of the ellipsoid inscribed in the tetrahedron, touching each face in the center of gravity and having its center at the center of gravity of the tetrahedron, are the roots of

$$k^6 - \frac{E^2}{2^4 . 3} k^4 + \frac{F^2}{2^4 . 3^3} k^2 - \frac{V^2}{2^6 . 3} = 0.$$

344. *By Prof. E. B. Seitz.*—Through each of two points, taken at random within a circle, a random chord is drawn; find (1) the prob'y that the chords will intersect; and (2) if a third random chord be drawn thro' a 3d random p't, find the prob's that the 3 chords will intersect in 0, 1, 2, 3 p'ts.

345. *By Prof. W. W. Johnson.*—The great circle from A (φ_1, λ_1) to B (φ_2, λ_2) passes north of the parallel of latitude φ_0; what is the longitude, λ_0 of the point P on this parallel so that the course APB shall be the shortest course from A to B which does not pass north of this parallel?

QUERY BY H. HEATON.—Is there any known general method of elimination when we have two or more equations containing two or more unk'n quantities, the equations being of the third degree or higher?

DETERMINATION OF A MERIDIAN.

BY W. L. MARCY, U. S. DEP. MIN'L SURVEYOR, LEADVILLE, COL.

THE simplicity of any independent calculation for the azimuth of Polaris, involving nothing but a few tables of an Ephemeris or Almanac and a common case in Spherical trigonometry, should not certainly have made the operation a forbidden field to the average surveyor. It is not so from want of ability. By practice he retains his knowledge of plane trigonometry, forgetting his spherical trigonometry for the want of its application.

A surveyor of more than ordinary accuracy and skill, sent from the far East to do some special work for a mining company, surprised me by expressing the opinion that the elongation of Polaris was equal to the complement of its declination in all latitudes. Another surveyor sits up all night to watch the star to its maximam elongation—a very accurate method when the latitude is closely known, but at so inconvenient a season that few really make the trial.

The observation of the star should not take over 5 to 15 minutes, and it can be made so early in the evening or late in the morning as to dispense

with artificial light to see cross-hairs, read verniers &c. I have caught Polaris in the field of view before sunset, and have seen it after sunrise, when in the field of view. To make these observations, the position of the star with respect to the pole must be found. The following approximation to its position will answer for bringing the star into the field of view:—

Represent the hour angle by H, the azimuth by A, the vertical distance above or below the pole by d, and the complement of its declination by c; then, approximately, $A=(c.\sin H)\div\cos l$, $d=c.\cos H$. Now, having the magnetic variation, increase it by A if the star is on the opposite side of the meridian to the needle, or the reverse, and raise the telescope on the vertical arc to $l\pm d$, according as d is above or below the pole, first adjusting the focus to the most distant object in sight. If the star is not in the field of view, the magnetic meridian being ariable, move the teles. slowly east and west until it is visible, then the rate and direction of motion will soon identify Polaris. The azimuth A, can be taken from the tables, by inspection, if the latitude is not too far removed. When the transit has no vertical arc a little more time and patience will generally succeed by slowly examining the sky in consecutive planes.

The principal errors in the meridian result from the inaccuracy of the time and latitude, but they would be much greater with other stars farther removed from the pole; the maximum error for one minute of time at the upper culmination of Polaris, in latitude 42° N., is 28″.7; and the max'm error for one degree of latitude is only 1′ 44″ or —1′ 38″.8, at the elongat'n.

To find the Hour Angle of Polaris.—If the observation is at evening, take the R. A. of mean sun, Greenwich mean noon for that day, but for the preceding day if the observation is in the morning; add to this the time of observation plus 12h, if in the morning, and also add the increase in R. A. corresponding to the time west from Greenwich and the time of observation, plus 12h, if in the morning. From these quantities deduct the apparent R. A. of Polaris for the day of observation if an Ephemeris is used, or the mean R. A. if an Almanac; the remainder, t^h, is the time the star is west ot the upper culmination, but the hour angle will be estimated from the upper or lower culmination as follows:—

For t^h negative Polaris is East above pole, hour ang. $= t^h\times15°$,
" $t^h<6^h$ and $>0^h$ Polaris is W. A. bole, $H=t^h\times15°$,
" $t^h<12^h$ " $>6^h$ " " W. B. " , $H=(12^h-t^h)\times15°$,
" $t^h<18^h$ " $>12^h$ " " E. B. " , $H=(t^h-12^h)\times15°$,
" $t^h<24^h$ " $>18^h$ " " E. A. " , $H=(24^h-t^h)\times15°$,
" $t^h<30^h$ " $>24^h$ " " W. A. " , $H=$ &c.

Having obtained the hour angle (H) of Polaris, take from the Ephemeris, for the day of observation, the complement of the apparent declination

and we shall have, with the co-lat., two sides and the included angle of a spherical triangle to find the azimuth, or angle opposite the co-declination, or the azimuth can be taken from the table and corrected for time and latitude by columns D and V.

If mean right ascension is used, the greatest error occurs at culmination, and if mean declination is used the greatest error occurs at elongation.

These errors increase with the lat. but in lat. 42° the first will probably never exceed 45″, and the second, 60″. The following numbers are the approximate corrections to be applied to the mean co-dec. of Polaris to obtain the apparent co-declination for the year 1882.

Jan, 1st, — 25″, Feb. 1st, — 25″, March 1st, —20″, April 1st, —11″, May 1st — 1″, June 1st, + 5″, July 1st, + 7″, Aug. 1st, + 3″, Sept. 1st, —4″, Oct. 1st, —15″, Nov. 1st, —26″, Dec. 1st, —36″, Dec. 31st, —42″.

Compared with previous years, the plus quantities are increasing and the minus decreasing slowly, approximately at the rate of 1″ in a year.

Inaccuracy in time produces a max. error at culmination, and in latitude, at elongation, which can be interpolated from the tables, and a meridian obtained from an assumed time and latitude can be corrected. If the assumed latitude and time are revised by azimuth of sun or star from the trial meridian, a second or third correction may be necessary to converge to the required accuracy.

If from trial meridian, by azimuth or meridian passage of the sun, our time is found to be not more than 15^m fast or slow, a second trial meridian from corrected time will give the time within a few seconds and the meridian sufficiently accurate for ordinary purposes.

The formulas $N67''.8(100 \pm 2.3) \div 100$, and $N75''.5(100 \pm 2.8) \div 100$ give the azimuth correction for one deg. of lat. either north or south of latitude 39° or 42°, respectively, within one second of arc, and they can be extended to 5° of latitude with error of less than 1′. The diff. for 1° of lat., increased or diminished, as we go north or south, .05, will be the diff. for the next degree, and so on to 5° with no error exceeding 2″.

The expression $A = c\,(\sin H) \div \cos l$, is a very convenient approximation to the azimuth, the max'm error, when $H = 45°$, not exceeding, in lat. 42°, 70″. Let e be the difference between the upper and lower azimuth when $H = 45°$, then $A = c\,(\sin H) \div \cos l \pm \frac{1}{2} e \sin 2H$ will approximate closely to the true azimuth, the + sign corresponding to above, and the — sign to below pole. This expression is more simple than the trigonometrical formula when two sides and the included angle are given, and gives an error of less than 2″ at lat. 45°, and less than 1″ at lat. 36°.

Example.—Polari is observed on the 15th of March, 1881, $6^h 45^m$, P. M., Lat. 41° N., Long. 105° W. = 7^h west of Greenwich.

R. A. mean sun =	$23^h\ 32^m\ 54^s.3$
Time of observation	6 45
Increase in R. A.	2 15 .5
	30 20 09 .8
Apparent R. A. of Polaris	1 14 31 .4
	29 05 38 .4
	—24
$H = 76°\ 24'\ 36'' =$	$5^h\ 05^m\ 38^s.4$ above and west of pole.

The apparent decl'n of Polaris is 88° 40′ 45″, therefore $c = 1°\ 19'\ 15''$. Using the letters as above, we have by Chauvenet's Trig., page 180,

$$\tan\varphi = \tan c \cos H, \qquad \cot A = \frac{\sin(90° - l \pm \varphi)\cot H}{\sin\varphi},$$

the upper sign for φ must be used when the star is below, and the reverse when above the pole. The calculation gives

$$\begin{array}{rlrl}
\tan c = & 8.3628023 & \cot H = & 9.3833498 \\
\cos H = & \underline{9.3710170} & \sin(90°-l-\varphi) = & \underline{9.8757224} \\
\tan\varphi = & 7.7338193, \ \varphi = 18'\ 38''.5 & & 9.2590722 \\
 & & \sin\varphi = & \underline{7.7338129} \\
A = & 1°\ 42'\ 32''.3. & \tan A = & 8.4747407
\end{array}$$

If we calculate A by the formula of approximation given above, we get

$$A = (c \sin H) \div \cos l + \tfrac{1}{2} e \sin 2H = 1°\ 42'\ 33''.2.$$

By the tables we have, interpolating 9″.8 for the 24′ 36″ in H,

for $H = 76°$, $\log N + \log c = 3.7951445 = \log 6239''.4$,	
for 24′ 36″	9 .8
Azimuth for $H = 76°\ 24'\ 36''$ in Lat. 42° N. =	6249″.2
Correction for one degree of lat., from Col. V,	—96 .4
	6152 .8
Correction for 1881, $2nd \div 100$,	—.3
$A = 1°\ 42'\ 32''.5 =$	6152″.5.

The most convenient approximation to the meridian is by the use of a Solar Compass, but the result is often crude and uncertain. Defective and poorly adjusted solars (and the adjustment is not very simple) may be found varying from 10′ to 30′, and an error of 3′ to 5′ in a fairly adjusted instrument is perhaps not to be considered extraordinary.

There may be no better method of securing greater accuracy than to adjust the solar to a meridian and watch the position of the image when in strict conformity to the same at different hours of the day.

The tables that are here presented are often not as convenient as direct calculation, but they can serve as guides, and the determinations for the 1st and 2nd of each month can not materially change in several years.

Table I.

	Factors for determ'ng the Az. of Polaris below Pole: Lat. 39° N.			Factors for determining the Az. of Polaris above Pole: Lat. 39° N.				
H.	Log N.	N.	M. Az.	Log N.	N.	M. Az.	D.	V.
1°	8.343375	0.02205	0° 1′46″	8.359718	0.02289	0° 1′50″	0″.4	1″.5
2	8.644345	0.04409	3 31	8.660678	0.04578	3 39		
3	8.820332	0.06612	5 17	8.836653	0.068 5	5 29		
4	8.945130	0.08813	7 02	8.961430	0.09150	7 18		
5	9.041891	0.11013	8 47	9.058080	0.11431	9 08		
6	9.120808	0.13207	10 33	9.137046	0.13710	10 57		
7	9.187471	0.15398	12 18	9.203691	0.15984	12 46		
8	9.245151	0.17585	14 02	9.261333	0.18253	14 34		
9	9.295948	0.19767	15 47	9.312090	0.20516	16 23		
10	9.341309	0.21943	17 31	9.357405	0.22772	18 11	4″.3	15″.4
11	9.382262	0.24114	19 15	9.398301	0.25021	19 59	4″.2	14″.9
12	9.419569	0.26277	20 59	9.435554	0.27262	21 46		
13	9.453809	0.28432	22 42	9.469732	0.28494	23 33		
14	9.485428	0.30579	24 25	9.501287	0.31717	25 19		
15	9.514781	0.32718	26 07	9.530566	0.33929	27 05		
16	9.542160	0.34847	27 49	9.557871	0.36130	28 51		
17	9.567795	0.36965	29 31	9.583421	0.38320	30 36		
18	9.591878	0.39073	31 12	9.607432	0.40498	32 20		
19	9.614588	0.41171	32 52	9.630042	0.42662	34 04		
20	9.636046	0.43256	34 32	9.651395	0.44812	35 47	8″.5	30″ 4
21	9.656368	0.45328	36 11	9.671627	0.46949	37 29	8″.2	29″.3
22	9.675667	0.47388	37 50	9.690811	0.49070	39 11		
23	9.694020	0.49433	39 28	9.709064	0.51176	40 52		
24	9 711503	0.51464	41 05	9.726442	0.53265	42 32		
25	9.728200	0.43481	42 42	9.743000	0.55335	44 11		
26	9.744153	0.55482	44 18	9.758836	0.57390	45 49		
27	9.759408	0.57466	45 53	9.773978	0.59426	47 27		
28	9.774045	0.59435	47 27	9.788475	0.61443	49 04		
29	9.788071	0.61386	49 01	9.802364	0.63440	50 39		
30	9.801536	0.63319	50 33	9.815689	0.65416	52 14	12″.4	44″.3
31	9.814474	0.65234	52 05	9.828480	0.67373	53 47	12″.0	42″.9
32	9.826909	0.67128	53 36	9.84075	0.69304	55 20		
33	9.83 867	0.69003	55 06	9.852579	0.71216	56 52		
34	9.850400	0.70860	56 35	9.863927	0.73102	58 22		
35	9.861495	0.72693	58 03	9.874883	0.74969	59 51		
36	9.872200	0.74507	59 30	9.885425	0.76811	1 01 20		
37	9.882525	0.76300	1 00 55	9.895576	0.78628	1 02 47		
38	9.892489	0.78071	1 02 20	9.905361	0.80420	1 04 13		
39	9. 02108	0.79819	1 03 44	9.914795	0.82186	1 05 38		
40	9 911379	0.81542	1 05 06	9.923896	0.83926	1 07 01	15″.9	56″.9
							15″.5	55″.3

TABLE I—CONTINUED.

H.	Log N.	N.	M. Az.	Log N.	N.	M. Az.	D.	V.
41°	9.920340	0.83242	1°06′28″	9.932670	0.85639	1°08′23″		
42	9.929000	0.84918	1 07 48	9.941140	0.87325	1 09 44		
43	9.937358	0.86568	1 09 08	9.949307	0.88983	1 11 03		
44	9 945440	0.88194	1 10 26	9.957205	0.90616	1 12 21		
45	9.953243	0.89793	1 11 42	9.964810	0.92217	1 13 38		
46	9.960796	0.91368	1 12 57	9.972154	0.93790	1 14 54		
47	9.968088	0.92916	1 14 11	9.979241	0.95333	1 16 08		
48	9.975140	0.94437	1 15 24	9.986070	0.96844	1 17 20		
49	9.981946	0.95928	1 16 36	9.992672	0.98327	1 18 31		
50	9,988525	0.97392	1 17 46	9.999034	0.99778	1 19 40	19″.0	67″.7
51	9.994882	0.98828	1 18 55	0.005169	1.01197	1 20 48	18″.5	66″.0
52	0.001020	1.00231	1 20 02	0.011078	1.02584	1 21 55		
53	0.006936	1.01610	1 21 08	0.016773	1.03938	1 23 00		
54	0.012666	1.02959	1 22 13	0.022269	1.05261	1 24 03		
55	0.018185	1.04276	1 23 16	0.027558	1.06551	1 25 05		
56	0.023528	1.05567	1 24 17	0.032663	1.07811	1 26 05		
57	0.028660	1.06822	1 25 18	0.037562	1.09034	1 27 04		
58	0.033607	1.08046	1 26 16	0.042266	1·10221	1 28 01		
59	0.038369	1.09237	1 27 13	0.046790	1.11376	1 28 56		
60	0.042956	1.10396	1 28 09	0.051123	1.12492	1 29 49	21″.4	76″.3
61	0.047365	1.11523	1 29 03	0.055280	1.13574	1 30 41	21″.0	74″.8
62	0.051596	1.12615	1 29 55	0.059270	1.14622	1 31 31		
63	0.055665	1.13675	1 30 46	0.063083	1.15633	1 32 20		
64	0.059570	1.14701	1 31 35	0.066730	1.16608	1 33 07		
65	0.063311	1.15694	1 32 23	0.070220	1.17549	1 33 52		
66	0.066890	1.16651	1 33 09	0.073540	1.18451	1 34 35		
67	0·070316	1.17575	1 33 53	0.076699	1.19316	1 35 16		
68	0.073587	1·18464	1 34 36	0.079705	1.20144	1 35 56		
69	0.076701	1·19316	1 35 16	0.082567	1.20939	1 36 34		
70	0.079667	1·20134	1 35 56	0.085267	1.21693	1 37 10	23″.1	82″.5
71	0.082483	1·20915	1 36 33	0.087803	1.22405	1 37 44	22″.8	81″.4
72	0.085153	1·21661	1 37 09	0.090203	1.23084	1 38 17		
73	0.087677	1.22370	1 37 43	0.092455	1.23724	1 38 48		
74	0·090058	1.23043	1 38 15	0.094560	1.24325	1 39 16		
75	0.092296	1.23678	1 38 45	0.096524	1.24888	1 39 43		
76	0.094392	1.24277	1 39 14	0.098347	1.25413	1 40 08		
77	0.096349	1.24838	1 39 41	0.100023	1.25898	1 40 32		
78	0.098167	1.25362	1 40 06	0.101560	1.26346	1 40 53		
79	0.099848	1.25848	1 40 29	0.102971	1.26758	1 41 13		
80	0.101389	1.26295	1 40 51	0.104236	1.27126	1 41 30	24″.2	86″.1
81	0.102799	1.26706	1 41 10	0.105353	1.27453	1 41 46	24″.0	85″.6
82	0.104071	1.27077	1 41 28	0.106342	1.27744	1 42 00		
83	0.105203	1.27410	1 41 44	0.107193	1.27995	1 42 12		

TABLE I—CONTINUED.

H.	Log N.	N.	M. Az.	Log N.	N.	M. Az.	D.	V.
84°	0.106206	1.27704	1° 41′58″	0.107916	1.28208	1°42′23″	24″.4	
85	0.107077	1.27961	1 42 11	0.108501	1.28381	1 42 31	24 .3	
86	0.107827	1.28182	1 42 21	0.108956	1.28516	1 42 37		
87	0.108428	1.28360	1 42 30	0.109274	1.28610	1 42 42		
88	0.108893	1.28497	1 42 36	0.109466	1.28667	1 42 45		
88	55′ 18″			0.109523	1.28684	Maximum 1 42 45	24 .4	
89	0.199235	1.28598	1 42 41	0.109521	1 28683	1 42 45		
90	0.109443	1.28660	1 42 44	0.109443	1.28660	1 42 44	24 .4	1′29″ 1 25

TABLE II.

	Factors for determ'ng the Az. of Polaris below Pole: Lat. 42° N.			Factors for determining the Az. of Polaris above Pole: Lat. 42° N.				
H.	Log N.	N.	M. Az.	Log N.	N.	M. Az.	D.	V.
1°	8.361910	0.02301	0° 1′51″	8.380080	0.02399	0° 1′55″	0″.4	1″.7
2	8.662876	0.04601	3 40	8.681041	0.04798	3 50		
3	8.838863	0.06900	5 30	8 857016	0.07195	5 45		
4	8 963661	0.09197	7 20	8.981794	0.09589	7 39		
5	9.070428	0.11493	9 10	9.078441	0.11980	9 34		
6	9.139400	0.13785	11 00	9.157398	0.14368	11 28		
7	9.206013	0.16070	12 50	9 224043	0.16751	13 22		
8	9.263693	0.18352	14 39	9.281685	0.19129	15 16		
9	9.314490	0.20630	16 28	9.332432	0.21500	17 10	4 .6	18 .3
10	9.359851	0.22901	18 17	9.377747	0.23864	19 03	4 .4	17 ·3
11	9.400815	0.25166	20 06	9 418641	0.26221	20 56		
12	9.438122	9 27423	21 54	9.455894	0.28569	20 49		
13	9.472362	0.29673	23 42	9.490072	0.30908	24 41		
14	9.503981	0.31914	25 29	9.521627	0.33237	26 33		
15	9.533347	0.34147	27 16	9 550894	0.35554	28 24		
16	9.560725	0.36368	29 02	9.578190	0.37861	30 14		
17	9.586370	0.38581	30 48	9.603747	0.40156	32 04		
18	9.610453	0.40781	32 34	9.627748	0.42437	33 53		
19	9.633163	0.42970	34 19	9.650358	0.44705	35 42	8 .9	35 .5
20	9.654632	0.45147	36 03	9.671699	0.46956	37 30	8 .5	31 .3
21	9.674969	0.47311	37 46	9.691926	0.49196	39 17		
22	9.694264	0.49461	39 29	9.711132	0 51420	41 03		
23	9.712617	0.51596	41 11	9.729357	0.53624	42 49		
24	9.730111	0.53717	42 53	9.746723	0.55811	44 34		
25	9.746820	0.55824	44 34	9.763280	0.57980	46 18		
26	9.662772	0.57913	46 14	9.779110	0.60133	48 01		

TABLE II—CONTINUED.

H.	Log N.	N.	M. Az.	Log N.	N.	M. Az.	D.	V.
27°	9.778050	0.59986	0° 47′54″	9.794245	0.62265	0° 49′42″		
28	9.792680	0.62041	49 32	9.808720	0.64376	51 24		
29	9.806712	0.64079	51 10	9.822609	0.66467	53 04		
30	9.820188	0.66098	52 47	9.835922	0.68537	54 43	13″0	51″.7
31	9.833138	0.68099	54 23	9.848701	0.70583	56 21	12 .6	49 .9
32	9.845562	0.70075	55 57	9.860980	0.72607	57 59		
33	9.857542	0.72035	57 31	9.872788	0.74608	59 35		
34	9.869080	0.73974	59 04	9.884145	0.76585	1 01 09		
35	9.880188	0.75891	1 00 36	9.895069	0.78536	1 12 42		
36	9.890900	0.77786	1 02 06	9.905640	0.80471	1 14 15		
37	9.901232	0.79659	1 03 36	9.915782	0.82373	1 15 46		
38	9.911208	0.81509	1 05 05	9.925558	0.84248	1 17 16		
39	9.920828	0.83335	1 06 33	9.934980	0.86096	1 18 45		
40	9.930120	0.85136	1 07 59	9.944054	0.87914	1 10 12	16 .7	66 .4
41	9.939092	0.86915	1 09 24	9.952797	0.87901	1 11 38	16 .2	64 .3
42	9.947765	0.88668	1 10 48	9.961255	0.91465	1 13 02		
43	9.956135	0.90393	1 12 11	9.969415	0.93200	1 14 25		
44	9.964221	0.92092	1 13 32	9.977300	0.94907	1 15 47		
45	9.972040	0.93765	1 14 52	9.984890	0.96584	1 17 07		
46	9.979600	0.95411	1 16 11	9.992229	0.98227	1 18 26		
47	9.986900	0.97029	1 17 29	9.999307	0.99841	1 19 43		
48	9.993955	0.98618	1 18 45	0.006130	1.01422	1 20 59		
49	0.000787	0.00181	1 20 00	0.012707	1.02969	1 22 13		
50	0.007389	1.01714	1 21 13	0.019060	1.04487	1 23 26	19 .8	78 .9
51	0.013744	1.03215	1 22 25	0.025179	1.05969	1 24 37	19 .3	76 .8
52	0.019894	1.04687	1 23 36	0.031074	1.07417	1 25 46		
53	0.025838	1.06130	1 24 45	0.036759	1.08833	1 26 54		
54	0.031563	1.07538	1 25 52	0.042243	1.10215	1 28 01		
55	0.037096	1.08917	1 26 58	0.047520	1.11563	1 29 05		
56	0.042458	1.10268	1 28 03	0.052611	1.12878	1 30 08		
57	0.047596	1.11582	1 29 06	0.057495	1.14155	1 31 09		
58	0.052558	1.12865	1 30 07	0.062178	1.15392	1 32 08		
59	0.057339	1.14112	1 31 07	0.066692	1.16598	1 33 06		
60	0.061924	1.15325	1 32 05	0.071011	1.17763	1 34 02	22 .4	88 .9
61	0.066349	1.16506	1 33 02	0.075163	1.18894	1 34 56	21 .9	87
62	0.070600	1.17652	1 33 57	0.079129	1.19985	1 35 48		
63	0.074683	1.18763	1 34 50	0.082923	1.21037	1 36 39		
64	0.078600	1.19839	1 35 42	0.086560	1.22056	1 37 28		
65	0.082344	1.20876	1 36 31	0.090034	1.23036	1 38 15		
66	0.085940	1.21882	1 37 19	0.093340	1.23976	1 38 60		
67	0.089384	1 22852	1 38 06	0.096494	1.24880	1 39 43		
68	0.092670	1.23785	1 38 51	0.099473	1.15739	1 40 24		
69	0.095800	1.24680	1 39 35	0.102308	1.26561	1 41 04		
70	0.098783	1.25540	1 40 14	0.104994	1.27348	1 41 41	24 .2	96 .1
							23 .9	94 .8

TABLE II--CONTINUED.

H.	Log N.	N.	M. Az.	Log N.	N.	M. Az.	D.	V.
71°	0.101611	1.26359	1° 40′54″	0.107527	1.28093	1°42 ′17″		
72	0.104296	1.27143	1 41 32	0.109911	1.28798	1 42 51		
73	0.106839	1.27890	1 42 07	0.112146	1.29462	1 43 23		
74	0.109232	1.28596	1 42 41	0.114237	1.30087	1 43 53		
75	0.111484	1.29265	1 43 13	0.116188	1.30673	1 44 20		
76	0.113598	1.29896	1 43 43	0.117994	1.31217	1 44 47		
77	0.115570	1.30487	1 44 12	0.119654	1.31720	1 45 11		
78	0.117404	1.31038	1 44 38	0.121179	1.32183	1 45 33		
79	0.119097	1.31551	1 45 03	0.122564	1.32605	1 45 53		
80	0.120655	1.32024	1 45 25	0.123810	1.32986	1 46 11	25″3	100″.4
81	0.122086	1.32456	1 45 46	0.124921	1.33327	1 46 28	25 .1	99 .7
82	0.123371	1.32852	1 46 05	0.125892	1.33625	1 46 42		
83	0.124523	1.33206	1 46 22	0.127431	1.33883	1 46 54		
84	0.125538	1.33518	1 46 38	0.127431	1.34101	1 47 5		
85	0.126419	1.33789	1· 46 50	0.128001	1.34277	1 47 13	25 .5	
86	0.127180	1.34023	1 47 01	0.128445	1.34414	1 47 20	25 .4	
87	0.127796	1.34214	1 47 10	0.128741	1.34506	1 47 24		
88	0.128286	1.34365	1 47 17	0.128917	1.34560	1 47 27		
88	48′ 5″			0.128964	1.34575	Maximum. 1 47 27	25 .6	
89	0.128642	1.34475	1 47 23	0.129857	1.34573	1 47 27		
90	0.123864	1.34544	1 47 26	0.128864	1.34544	1 47 26	25 .6	1′.44″ 1′.39″

Explanation of Tables.—Tables I and II give the mean azimuth of polaris for the epoch 1880, in latitude 39° and latitude 42°, north, respectively, corresponding with the value of H (the hour angle) as indicated in 1st column. The mean azimuth, as given in the 4th and 7th col's, is the product of the factor N multiplied by the co-declination (c) of Polaris, the factor N being the equivalent of the formula $\sin H \div \sin(l \pm \cos H \times c)$. The azimuth thus determined is slightly in excess but less than 1″ at 45° and less than 2″ at 90°, and for values of H greater than 30° I have reduced N to correspond with more exact calculations. Column D contains a correction for annual variation in azimuth and is always *minus*, and column V, a correction for 1° of latitude, + when north and — when south. The double numbers in columns D and V correspond to the angle above and below pole, respectively; the upper in column V to be applied with the plus, and the lower, with the minus sign. If d be the difference between the upper and lower numbers, in column, D for any value of H in 1880, then, for any number n, of years after that epoch, the correction will be $2nd \div 100$, nearly.

When polaris is moving towards its culmination, positive increase in longitude and time diminishes its azimuth in table III, and vice versa.

TABLE III.

TIME FOR OBSERVATION ON THE FIRST AND SECOND DAY OF EACH MONTH.	Special determinations of the azimuth of Polaris, the apparent Dec. and R. A. being used. Lat. 39 deg. N., Long. 106 deg. W.				Special determinations of the azimuth of Polaris, the apparent Dec. and R. A. being used, Lat. 42 deg. N., Long. 106 deg. W.				POSITION OF POLARIS WITH RESPECT TO POLE, A=ABOVE; B=BELOW; E=EAST; W=WEST OF POLE. WHEN POLARIS IS A. E. OR B. W. IT IS DECREASING WITH POSITIVE TIME IN AZIMUTH.	Approx. am't to be added or subtracted from the Lat. to give the Alt. of Polaris.
	Azim'th for 1881.	Azim'th for 1882.	V'n of Azim. for 1 m, inc. is pos.	V'n of Azim. for 15° long., w is pos.	Azim'th for 1881.	Azim'th for 1882.	V'n of Azim. for 1 m, inc. is pos.	V'n of Azim. for 15° long., w is pos.		
	° ′ ″	° ′ ″	″	″	° ′ ″	° ′ ″	″	″		
Jan. 1, 5h 00m 00.0s p. m.	0 38 58	0 39 26	—25.0	—04.1	0 40 50	0 41 19	—26.0	—04.2	E. A. Pole, moving west	+01 13
Jan. 2, 7h 00m 00.0s a. m.	0 14 43	0 14 02	+26.0	+04.3	0 15 23	0 14 39	+27.0	+04.4	E. B. Pole, moving east	—01 18
Feb. 1, 5h 30m 00.0s p. m.	0 28 45	0 28 00	+26.0	+04.3	0 30 08	0 29 21	+27.5	+04.5	A. W. Pole, moving west	+01 16
Feb. 2, 6h 50m 00.0s a. m.	0 59 51	0 59 06	+21.0	+03.4	1 00 59	1 00 12	+22.5	+03.6	B. E. Pole, moving east	—01 04
Mar. 1, 6h 15m 00.0s p. m.	1 24 26	1 22 46	+15.0	+02.5	1 28 24	1 27 42	+16.5	+02.6	W. A. Pole, moving west	+00 46
Mar. 2, 6h 00m 00.0s a. m.	1 19 07	1 18 27	+16.7	+02.7	1 22 39	1 21 57	+17.7	+02.8	E. B. Pole, moving east	—00 49
April 1, 6h 45m 00.0s p. m*	1 41 53	1 41 28	—02.0	—00.3	1 46 32	1 46 06	—02.0	—00.3	W. B. Pole, mo'ing e. fr'm elong.	—00 04
April 2, 5h 20m 00.0s a. m.	1 36 27	1 36 09	+09.0	+01.5	1 40 48	1 40 29	+09.4	+01.5	E. B. Pole, moving east	—00 19
May 1, 7h 30m 00.0s a. m.	1 12 26	1 12 47	—19.0	—03.1	1 15 39	1 16 02	—19.7	—03.2	W. B. Pole, moving east	—00 55
May 2, 4h 30m 00.0s a. m.	1 42 14	1 41 50	+00.4	+00.0	1 46 55	1 46 30	+00.4	+00.0	E. B. Pole, mo'ing east to elong.	—00 01
			Near E	Elong.			Near E	Elong.		
June 1, 8h 00m 00.0s p. m.	0 14 00	0 14 35	—26.1	—04.3	0 14 37	0 15 13	—27.0	—04.4	W. B. Pole, moving east	—01 18
June 2, 4h 00m 00.0s a. m.	1 35 25	1 35 18	—10.5	—01.7	1 39 52	1 39 44	—10.5	—01.7	E. A. Pole, moving west	+00 30
July 1, 8h 00m 00.0s p. m.	0 36 48	0 36 05	+24.7	+04.0	0 38 24	0 37 39	+25.7	+04.2	E. B. Pole, moving east	—01 14
July 2, 4h 10m 00.0s a. m.	1 00 44	1 01 03	—22.2	—03.6	1 03 37	1 03 57	—23.0	—03.8	E. A. Pole, moving west	+01 04
Aug. 1, 7h 30m 00.0s p. m.	1 10 37	1 09 55	+19.0	+03.1	1 13 44	1 13 00	+20.0	+03.3	E. B. Pole, moving east	—00 57
Aug. 2, 4h 30m 00.0s a. m.	0 00 30	0 01 09	—27.5	—04.5	0 00 32	0 01 12	—28.7	—04.7	E. A Pole, moving west	+01 19
Aug. 2, 4h 31m 07.4s a. m.	†		+27.5	+04.5	†		+28.7	+04.7	A. Pole, moving west	+01 19
Aug. 2, 4h 32m 31.4s a. m.		†	+27.5	+04.5		†	+28.7	+04.7	A. Pole, moving west	+01 19
Sept. 1, 7h 00m 00.0s p. m.	1 33 36	1 33 00	+10.6	+01.7	1 37 48	1 37 11	+11.2	+01.8	E. B. Pole, moving east	—00 39
Sept. 2, 5h 00m 00.0s a. m.	1 01 35	1 00 53	+22.0	+03.6	1 06 27	1 05 44	+22.5	+03.7	W. A. Pole, moving west	+01 03
Oct. 1, 6h 15m 00.0s p. m.	1 41 30	1 41 05	+02.6	+00.4	1 46 08	1 45 41	+02.8	+00.4	E. B. Pole, moving east	—00 06
Oct. 2, 5h 15m 00.0s a. m.	1 37 00	1 36 27	+08.6	+01.4	1 41 30	1 40 56	+08.8	+01.4	W. A. Pole, moving west	+00 26
Nov. 1, 5h 30m 00.0s p. m.	1 38 49	1 38 38	—06.3	—01.0	1 43 24	1 43 12	—06.5	—01.1	E. A. Pole, moving west	+00 20
Nov. 2, 6h 00m 00.0s a. m.	1 33 37	1 33 03	—10.6	—01.7	1 37 16	1 37 13	—11.2	—01.8	W. B. Pole, moving east	—00 31
Dec. 1, 5h 00m 00.0s p. m.	1 22 08	1 22 12	—16.2	—02.7	1 27 59	1 25 04	—16.7	—02.7	E. A. Pole, moving west	+00 47
Dec. 2, 6h 40m 00.0s a. m.	0 00 11	0 46 35	—23.5	—03.8	0 48 13	0 48 38	—24.5	—04.0	W. B. Pole, moving east	—01 10

* Near Western Elongation.　　　† Upper Culmination.